DARK MATTER: THE PHYSICS BEYOND STANDARD MODEL

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PH12m1010

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A Dissertation Submitted to The Faculty of the DEPARTMENT OF PHYSICS In Partial Fulfillment of the Requirements For the Degree of Master of Sciences at Indian Institute Of technology Hyderabad

May 2, 2014

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Declaration

I declare that this written submission represents my ideas in my own words, and where ideas or words of others have been included , I have adequately cited and referenced the original sources.I also declare that I have adhered to all principles of acedamic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission.

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Acknowledgement

I owe my profound gratitude to my supervisor Dr. Narendra Sahu. Without his unwavering guidance and persistent help this dissertation would never have been possible.
I would like to thank our HOD Dr. Anjan Giri for his kind cooperation.
I would also like to thank PhD scholars G.Sweta and Nirakar Sahoo for helping me out of some computer related issues.
Also a big thank to all of my classmates for their support and encouragement.

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1 Abstract

Despite the enormous success of Standard Model in describing the fundamental interactions of elementary particles, there are certain issues need to be addressed. One of them is the relic abundance of "Dark Matter". Cosmological observations imply that there exist enormous amount of "Dark Matter" in the Universe. In this article first we study how the relic abundance of cold dark matter (CDM) is created in the early universe. Then we extend the standard model (SM) to incorporate a particle physics candidate of dark matter. In particular, we extend the SM by a real scalar singlet S and impose a discrete Z_2 symmetry to make it stable. Here we have assumed the mass of Dark Matter candidate S to be in the range 1GeV $< m_D <$ 1TeV and Higgs mass $m_h = 126$GeV as recently quantified by CMS and ATLAS experiments at CERN. Then using the Boltzmann equation we calculate the relic density of S. We obtain the relevant parameter space for which S is a good candidate of dark matter.

2 The Standard Model

The fundamental question- "what is matter made of? " has always been bothered scientists down the decades.The theories and discoveries of thousands of physicists since 1930 have resulted in a remarkable insight in fundamental structure of matter. Everything in the universe is found to be made up of some basic building blocks governed by four fundamental forces. Our best understanding of how these particles and forces are related to each other is encapsulated in the standard model (SM) of particle physics. Developed in early 1970s, it has successfully explained almost all experimental results and predicted accurately a wide varieties of physical phenomena.

Matter Particles : All matter around us is made of two types of elementary particles- quarks and leptons. Each group consists of six particles, which are related in pairs or "generations". The lightest and stable particles fall into 1st generation, while the heavier and unstable particles belong to 2nd and 3rd generations. Quarks come in six "flavors" and also in three "colors".

Forces and Carrier particles: There are four fundamental forces in the universe. They are strong, weak, electromagnetic and gravity. These fundamental forces result from exchange of force-carrier particles, which belong to a broader group called "boson". Strong force is carried by "gluons", electromagnetic is by "photon", weak interaction is by W and Z bosons and yet not found graviton should be responsible for gravitational force. We ignore gravitational interaction due to its strength is much much smaller than the other fundamental forces.

At one time electricity and magnetism were thought to be distinct form of forces. But based on the observations of Oersted and Faraday, Maxwell formulated the theory of electromagnetism which combines electricity and magnetism as one form of force. Einstein dreamed of going a further step, combining gravity with electrodynamics in a single unified theory. Although his attempt failed, but it inspired Glashow, Weinberg and Salam (GWS) to unify the weak and electromagnetic forces known as "electroweak" force. This unification implies electricity, magnetism, light and some types of radioactivity are all manifestations of this single force at high energy. And the gauge symmetry underlying is called $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$, where $\mathrm{SU(2)_L}$ refers to the weak isospin involving only left handed component of SM particles and $\mathrm{U(1)_Y}$ refers to weak hypercharge involving both left and right handed particles of SM. [8] This theory is associated with four mediating particles-γ, W^+, W^-, Z^0. But there was a major flaw in this theory. If we demand our theory to be invariant under local gauge transformation:$\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$, which is a symmetry of the system, then we have to add some gauge fields to the Lagrangian in order to preserve its form and also these gauge bosons have to be massless as governed by this symmetry. Though it is true for photon but not for $W^{\pm}, Z^0$, whose observed masses are 80 and 90 GeV respectively. Fortunately in 1964 Robert Brout, Francois Englart and Peter Higgs proposed a mechanism better known as "Higgs Mechanism" to solve the mass issue, which was adopted in GWS model. According to "Higgs mechanism" all the gauge bosons acquire masses through their interaction with a scalar field called "Higgs field' , which has a potential

$$V = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \, . \tag{1}$$

when the Higgs acquires a vacuum expectation value (VEV), the gauge symmetry $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ breaks spontaneously to $U(1)_{\rm em}$. As a result three of the gauge bosons: $W^{\pm}, Z^0$ acquire masses while the photon remain massless.[1]

Not only the gauge bosons, all the quarks and lepton masses were also not allowed in the SM Lagrangian as they break gauge symmetry. They acquire their masses through interaction with the Higgs via Yukawa coupling. Thus the "Higgs Boson"(quanta of Higgs field) is indispensable in SM, as it gives masses to all other particles. But for many years experimental evidence of Higgs Boson was a mirage and therefore people began to doubt the validity of the SM. In 2013 the ATLAS and CMS experiments at CERN's Large Hadron Collider announced the existence of a particle having energy 126 Gev which is consistent with the Higgs Boson.

The discovery of Higgs Boson completes the SM and it is the best description for subatomic world as of now. But this theory explains only about 4% of the total energy budget of the Universe. The remaining 95% of the total mass content of our Universe is made of a form of unknown matter which can not be understood within the particle content of the SM. For example, it can not explain the neutrino mass problem. Neutrinos do not have mass in the SM, but experiments like solar and atmospheric neutrino oscillations demand that they should be massive. Another aspect is the "Dark Matter" which constitutes about 27% of the total energy budget of thye Universe. But the SM does not have any candidate of dark matter.

2.1 Dark Matter : The Missing Mass Of The Universe

One of the most important and demanding aspect of modern cosmology is to probe the existence and characteristics of particle dark matter. The observations by Wilkinson Microwave Anisotropy Probe or WMAP [6]for studying the fluctuations in cosmic microwave background reveals that, the universe consists of around 32% matter and rest around 68% is unknown dark energy. Out of this 32% matter only about 5.2% accounts for ordinary matter consisting of leptons, baryons which make stars, galaxies *etc.*. The rest 26.8% are completely unknown, they can not be directly seen with telescopes, evidently they neither emit nor absorb light or other electromagnetic spectra at any significant level. This huge amount of unseen matter, contributing more than 90% of total matter content of the universe is known as Dark Matter. Different cosmic microwave anisotropy measurements predict baryonic density to be $\Omega_b \approx 0.04$, which is far less than the total dark matter density $\Omega_{DM} \approx 0.23$. This indicates the fact that most of the Dark Matter in the universe are non baryonic in nature.

2.2 Evidences

In 1933 swiss astronomer Fritz Zwicky had first suggested the possibility of dark matter. He observed that the rotational velocities of spiral galaxies required much more mass than was present in the form of matter. Let us suppose v is the velocity of a star in spiral galaxy and r is the distance of that star from the center of the galaxy. Gravitational force will provide the centripetal force and we get

$$GM(r) = rv^2 \tag{2}$$

where $M(r)$ is the mass interior to r. Now taking r to be the distance within which most of the light emitted by the galaxy (i.e most of the visible mass is concentrated), then

$$M(r) = \frac{4}{3}\pi r^3 \rho \tag{3}$$

where ρ is the mass density assumed to be constant.Then from Eq. 2 we get $v \propto r$. If the mass associated with light was the whole story then after this distance M would have been constant and we expect again from Eq. 2 $v \propto r^{-\frac{1}{2}}$. But what was actually observed that v remains almost constant with radial distance.Then again from Eq. 2 we get $M \propto r$. This unknown amount of mass was named "dark" as there was no radiation associated with it. Another important evidence for Dark Matter comes through Gravitational Lensing. While

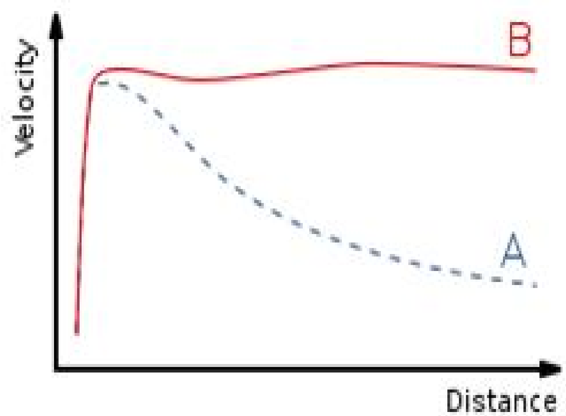

Figure 1: Rotational velocity curve for spiral galaxies [2]

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coming from a distant source to the observer if light passes through a massive object then its path bends due to the distorted space-time curve. And the original source is observed as a ring around the lensing object known as Einstein ring. This effect is known as Gravitational Lensing. In many galaxies the lensing effect found is much greater than the luminous mass, this strongly supports the existence of Dark Matter [10].

Bullet cluster also show signature of Dark Matter. It consists of two colliding clusters of galaxies.The hot gas of the two colliding components, seen in X-rays represents most of the mass of baryonic matter in cluster pair. The Dark Matter was detected indirectly through Gravitational Lensing. The lensing is strongest in two separated regions.This supports for the idea that most of the mass in the cluster pair is in the form of collision less dark matter [9].

Based on the evidences we can claim that Dark Matter should have the following properties 1. Adequately massive so they must move with non relativistic velocity. Therefore called Cold Dark Matter or CDM.
2. They have only gravitational interaction.
3. They should be stable or very long lived, otherwise large scale structure would not have

been possible.
4.They Should be electrically and color neutral.
5.Mostly non-baryonic in nature.
In addition to that the CDM is assumed to have weak interaction. Therefore, it is also called weakly interacting massive particle (WIMP). Because of weak interaction it can be detected directly in the terrestrial laboratories. In order to study the details of dark matter we need to know the thermodynamics of early universe which we discuss in the following section.

3 COSMOLOGICAL MODEL

Observations describe our universe is approximately homogeneous and isotropic on large scale, i.e. geometry of space-time is spherically symmetric about any one point in space (isotropic) and same at any point (homogeneous). These are the basic assumptions of Friedmann-Robertson-Walker(FRW) Model. Geometry of space-time in FRW model is best described by the metric

$$ds^2 = -dt^2 + a(t)^2[dr^2/1 - kr^2 + r^2(d\theta^2 + \sin\theta^2 d\phi^2) \tag{4}$$

where k=1,0,-1 for closed, flat and open universe respectively and a(t) is the scale factor denoting the growth of co-ordinate with expansion of the universe. The FRW model assumes the universe as a cosmological fluid consisting of three noninteracting components-
Pressureless matter- Mostly consisting of gas-dust of the galaxies. Their thermal energy is much less than the rest energy, so they can be well approximated by pressureless matter.

Radiation- Radiation includes the cosmic ray background photons as well as neutrino species with zero or small rest mass moving relativistically.

Vacuum- Characteristics of the vacuum are unknown and we assume it to be constant. [3]

3.1 First law Of Thermodynamics

We assume that the universe is adiabatically expanding by spending its own energy. Heat flow in any direction is not allowed, otherwise it will violate the assumption of isotropy. So the first law of thermodynamics for a cosmological fluid in a volume ΔV connects infinitesimal changes in that volume $\delta(\Delta V)$ to a corresponding infinitesimal change in its total energy $\Delta(\delta E)$, namely

$$\delta(\Delta E) = -P\delta(\Delta V) \tag{5}$$

Here, P is the pressure exerted by matter in the physical volume $\Delta(\delta V)$, ΔE is $\rho(\delta V)$, where ρ is the total energy density. In terms of co-moving volume we write $\Delta V = a^3(t)\Delta V$ then the first law takes the form

$$\frac{d}{dt}\rho a^3(t) = -P\frac{d}{dt}a^3(t) \tag{6}$$

Matter : For the pressureless matter $P = 0$ $\frac{d}{dt}\rho_m a^3(t) = 0$
then, from Eq.6

$$\rho_m(t) = \rho_m(t)[a(t_0)/a(t_0)]^3 \tag{7}$$

t_0 is the present instant of time. Thus the overall time dependence of matter density is determined by the scale factor a(t)

Radiation: For a gas of black body radiation at temperature T energy density $P_r = \frac{\rho_r}{3}$. Putting this into Eq.6 we get

$$\rho_r(t) \quad = \quad \rho_r(t_0)[a(t_0)/a_(t)]^4 \tag{8}$$

Vacuum: Nature of vacuum energy is still not predicted. Generally it is assumed that vacuum energy is: 1. constant in space and time; 2. positive as indicated by present observations. Thus the first law of thermodynamics implies,
$P_v = -\rho_v$

3.2 Dynamics Of FRW Model: Friedmann Equation

The dynamics of expanding universe only appears implicitly in the time dependence of scale factor a(t). To make the time dependence implicit one has to solve the Einstein Equation of General Relativity.The main essence of General Relativity is that the presence of matter produces space time curvature, so in schematic form
(measure of local space time curvature) = (measure of matter energy density)
This relation is called Einstein equation and has the mathematical form

$$R_{\mu\nu} - \frac{1}{2}\mathcal{R}_{\mu\nu} \quad = \quad 8\pi G T_{\mu\nu} \tag{9}$$

$R_{\mu\nu}$ is Ricci tensor, $\mathcal{R}$ is the Ricci scalar(see appendix A) and $T_{\mu\nu}$ is the stress-energy tensor for all the fields present in the form of matter and radiation. To be consistent with the symmetry of Robertson-Walker metric defined in Eq. 4, $T_{\mu\nu}$ is required to be diagonal and by isotropy the spatial components of $T_{\mu\nu}$ are required to be equal.Thus

$T_{\mu\nu} = \text{diag}(\rho, -P, -P, -P)$. The $0-0$ component of Einstein equation is

$$\dot{a}^2/a^2 + k/a^2 \;=\; 8\pi G\rho/3 \tag{10}$$

This equation is called Friedmann equation. For flat space (i.e., k=0), the above equation becomes $\dot{a}^2/a^2 = 8\pi G\rho/3$. Using $\dot{a}/a = H$, the Hubble constant, we can write

$$\rho \;=\; \frac{3H^2}{8\pi G} \tag{11}$$

where ρ defines the energy density of universe at a Hubble scale H. At present $H = H_0$ and hence $\rho = \rho_0 \equiv \rho_c$, called critical energy density of the Universe. At any instant the relative fractions of matter, radiation and vacuum energy can be calculated by dividing them with ρ_c. The relative fractions at present are conventionally denoted by $\Omega_m = \rho_m(t_0)/\rho_c$, $\Omega_r = \rho_r(t_0)/\rho_c$ $\Omega_v = \rho_v(t_0)/\rho_c$. As a result one can define: $\Omega_m + \Omega_r + \Omega_v = 1$ The early universe was mostly radiation dominated. But as the universe expands and cools down gradually matter takes over radiation and now it is mostly matter dominated. At the matter dominated epoch $\rho_{tot} \approx \rho_m$ Sine $\rho_r \propto 1/a^3(t)$; using Friedmann equation we see $a \propto t^{2/3}$. Similarly for radiation dominated epoch $\rho_{tot} \approx \rho_r$ and $a \propto t^{1/2}$. For the vacuum dominated epoch $\rho_{tot} \approx \rho_v$ and $a \propto \exp Ht$.

It is clear that in all the three cases the universe expands with time. In the matter and radiation dominated cases, universe begins with a singularity with $a = 0$ at $t = 0$, all the physical quantities like energy density, temperature become infinite then. The moment $t = 0$ is called the The Big Bang. But for the vacuum dominated case $a = 0$ at $t = -\infty$. Whether this is a singularity or not is not clear since the vacuum energy ρ_v has always been constant. Experimental evidences show we now live in somewhere between matter and vacuum dominated region.

4 Thermodynamics Of Early universe

Today the radiation in the universe is comprised of 2.75 K microwave photons and three 1.96K relic neutrinos, which move relativistically. In the early universe all particles were in thermal bath, so that they can be well approximated as to be in thermal equilibrium. And there were significant amount of relativistic particles present. Here we will calculate various thermodynamic quantities like number density(n), energy density(ρ), pressure(P) in terms of chemical potential(μ), degrees of freedom(g) and temperature (T).

In the ultra relativistic limit($T >> m$) and for ($T >> \mu$) the relations are:

$$\rho \;=\; \frac{\pi^2}{30} g T^4 (\text{for bosons}) \tag{12}$$

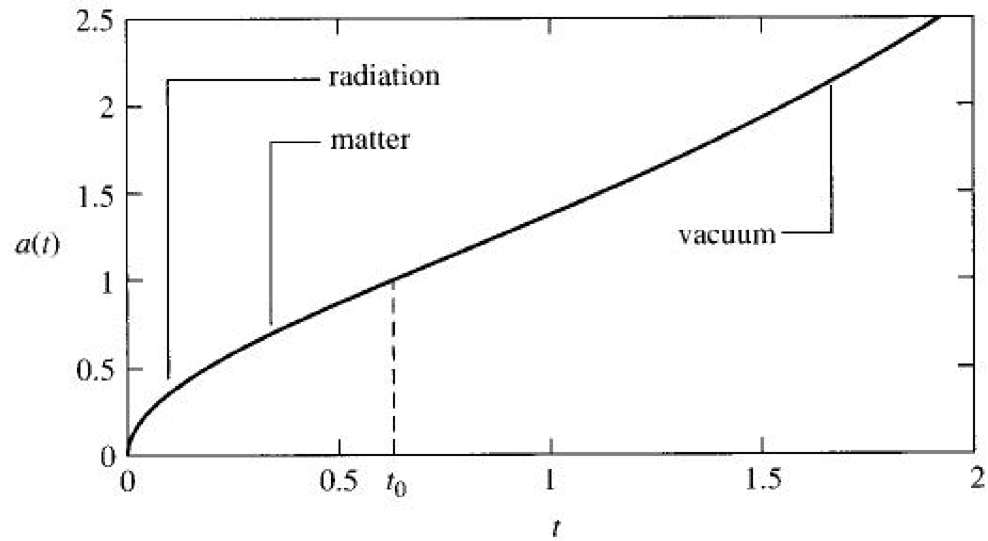

Figure 2: evolution of scale factor with time in various epochs in flat FRW model. [3]

$$\begin{aligned}
&= \frac{7}{8}\frac{\pi^2}{30} g T^4 (\text{for fermions}) \\
n &= \frac{\zeta(3)}{\pi^2} g T^3 (\text{for bosons}) \qquad (13)\\
&= \frac{3}{4}\frac{\zeta(3)}{\pi^2} g T^3 (\text{for fermions}) \qquad (14)\\
P &= \rho/3 \qquad (15)
\end{aligned}$$

The total energy density contributed by all the relativistic species together can be expressed as,

$$\begin{aligned}
\rho_{total} &= \frac{\pi^2}{30} \sum_{i=bosons} g_i T_i^4 + \frac{7}{8}\frac{\pi^2}{30} \sum_{i=fermions} g_i T_i^4 \qquad (16)\\
&= g_* \frac{\pi^2}{30} T^4 \qquad (17)
\end{aligned}$$

Here we have considered the probability that the i-th species may have a thermal distribution but with different temperature than that of photon. Here g_* counts the total no of effective massless degrees of freedom defined by:

$$g_* = \sum_{i=bosons} g_i (T_i/T)^4 + \frac{7}{8} \sum_{i=fermions} g_i (T_i/T)^4 \qquad (18)$$

For $T \geq 300$GeV (i.e the temperature for electroweak phase transition) all species in the Standard Model- 8 gluons, W^+, W^-, Z^0, 3 generations of quarks and leptons and 1 complex Higgs Doublet should have been relativistic, with $g_* = 106.75$. In non-relativistic limit ($m >> T$) the number density, energy density and pressure are same for Bose and fermion species. The details of analytical calculation can be found in appendix-B.

$$
\begin{aligned}
n = g(\frac{mT}{2\pi})^2 \exp[-(m-\mu)/T] \\
\rho = mn \\
P = nT
\end{aligned} \tag{19}
$$

4.1 Entropy

In the early universe mostly the reaction rates of particles in thermal bath, τ_{int} were much greater than the expansion rate of the universe H, and the local thermal equilibrium was sustained. Here in this section we will concentrate on the entropy of the expanding universe.

In the expanding universe the second law of thermodynamics applied to a comoving volume element of unit co-ordinate volume element and physical volume $V = R^3$, implies that

$$
Tds \quad = \quad d(\rho V) + PdV = d[(P+\rho)V] - VdP \tag{20}
$$

Using thermodynamic relations it can be shown that in thermal equilibrium entropy $S = (P+\rho)V/3$ is conserved. The details of calculation can be found in Appendix-C. The entropy density s is defined as

$$
s = S/V = (P+\rho)/3 \tag{21}
$$

The entropy density is dominated by the contribution of relativistic particles. Using Eqs. 13 and 15 we get

$$
s \quad = \quad \frac{2\pi^2}{45} g_{*s} T^3 \tag{22}
$$

Since through most of the history of early universe all particles maintained the thermal equilibrium, thus had a common temperature. Therefore, g_{*s} can be replaced by g_*. In todays universe, g_* and g_{*s} have different values. Because in the present universe only relativistic particles species are photons and 3 neutrinos having different temperatures. Relation between neutrino temperature T_ν and photon temperature T is $T/T_\nu = (11/4)^{1/3} = 1.40$. Using the above relation and putting into Eq. 18, we can find $g_* = 3.36$ and $g_{*s} = 3.91$.

Since S is conserved $s \propto a^{-3}$, and therefore $g_{*s}T^3a^3$ remains constant as universe expands. The number of a species in a co moving volume $N = a^3 n$ is equal to the number density of that species divided by entropy density, i.e., $N = n/s$. Using formulas for n and s, we find

$$N = \frac{45\xi(3)g_*}{2\pi^4 g_{*s}}(forT >> m) \tag{23}$$

$$= \frac{45g_*}{4\sqrt{2}\pi^5 g_{*s}}(m/T)^{3/2}\exp[-(m-\mu)/T](forT << m) \tag{24}$$

From the above two relations we can comment that for highly relativistic species there is equilibrium abundance but for massive species the abundance is small due to Boltzmann suppression term $\exp[-m/T + \mu/T]$.

5 Boltzmann Equation

Most of the period in early universe the particles were in thermal equilibrium as long as their reaction rate τ was greater than the expansion rate (Hubble) of the universe. Soon after $\tau \leq H$ the species gets decoupled form the thermal plasma and remained as relics. This phenomena is called "freeze out" and the temperature at which this freeze out occurs is called "freeze out temperature". In this section we will discuss about the "Boltzmann equation" which is an useful tool for studying the properties of decoupled species. The equation has the generic form $\hat{L}[f] = C[f]$

Where [f] is the phase space distribution function of the particle , $\hat{L}$ is the Liouville operator and C is the collision operator. The relativistic generalization of Liouville operator is

$$\hat{L} = p^\alpha \frac{\partial}{\partial x^\alpha} - \tau^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha} \tag{25}$$

For FRW model the phase-space density is spatially homogeneous and isotropic,i.e $f = f(|\vec{p}, t| = f(E,t))$. For Robertson-Walker metric as defined by 4, $\hat{L}$ operator is

$$\hat{L}f(E,t) = E\frac{\partial f}{\partial t} - \frac{\dot{a}}{a}|\vec{p}|^2 \frac{\partial f}{\partial E}. \tag{26}$$

Using the definition of number density in terms of phase space density

$$n(t) = \frac{g}{2\pi^3}\int d^3p f(E,t)$$

$$\frac{dn}{dt} + 3\frac{\dot{a}}{a}n = \frac{g}{2\pi^3}\int C[f]\frac{d^3p}{E} \tag{27}$$

The details can be found in Appendix-D.

The collision term for the process $a + b \leftrightarrow i + j$ is given by:

$$\frac{g}{2\pi^3}\int C[f]\frac{d^3p}{E} = -\int d\Pi_a d\Pi_b d\Pi_i d\Pi_j (2\pi)^4 \times$$
$$\delta^4(p_a + p_b - p_c - p_d) \times [|M|^2_{a+b\leftrightarrow i+j} f_a f_b (1 \pm f_i)(1 \pm f_j) - |M|^2_{i+j\leftrightarrow a+b} f_i f_j (1 \pm f_a)(1 \pm f_b)] \quad (28)$$

Here $1 - f$ is the fermi blocking factor. Creation of particle in a final momentum p will be hindered due to pauli exclusion principle. Similarly $1 + f$ is the stimulated factor for bosons. It denotes creation of a boson with momentum P in final state is encouraged.

Now we assume -

1. CP or T invariance; which suggests equivalence of forward and backward reaction,i.e $|M|^2_{a+b\leftrightarrow i+j} = |M|^2_{i+j\leftrightarrow a+b}$

2. absence of B.E condensation or fermi degeneracy, so that we can replace $1 \pm f$ as f since f is very small compared to 1.

Now, we want to scale out the effect of the expansion of the universe by considering the number of particles in a co moving volume, defining $Y = \frac{n}{s}$ and using the fact that $sa^3 = constant$ we get

$$\dot{n} + 3Hn = s\dot{Y} . \quad (29)$$

Defining $x = m/T$ we can write the above Boltzmann equation as

$$\frac{dY}{dx} = -\frac{x}{H(m)s}\int d\Pi_a d\Pi_b d\Pi_i d\Pi_j (2\pi)^4 \times \delta^4(p_a + p_b - p_c - p_d)$$
$$\times |M|^2 (f_a f_b - f_i f_j) \quad (30)$$

Where $H(m) = 1.66 g_*^{1/2}\frac{m^2}{m_{PL}}$, m_{PL} being Planck mass. The details of analytical calculation can be found in Appendix- E.

5.1 Freeze Out:Relic Abundance

As we have discussed above for a massive particle remaining in thermal equilibrium has abundance $n/s \approx (m/T)^{3/2} \exp -(m/T)$ in the limit $m/T >> 1$ and hence negligible due to the exponential term. If the interactions of the species freeze out (i.e $\tau \leq H$) at a temperature at which m/T is not greater than 1, the species can have a significant relic abundance today. To calculate the relic abundance let us assume that a species ψ is stable, so that only annihilation and inverse annihilation process like $\psi\overline{\psi} \leftrightarrow X\overline{X}$ can change the number of ψ and $\overline{\psi}$. Even if ψ is out of equilibrium after decoupling, X 's are still in thermal equilibrium.

Now let us consider the the factor $[f_\psi f_{\overline{\psi}} - f_X f_{\overline{X}}]$ in the collision term in Boltzmann equation; since $X, \overline{X}$ are in thermal equilibrium and assuming zero chemical potential $f(x) = \exp -E_X/T$, $f(x) = \exp -E_{\overline{X}}/T$. The delta function in Boltzmann Eq. 30 demands the conservation of energy: $E_\psi + E_{\overline{\psi}} = E_X + E_{\overline{X}}$, so that $f_X f_{\overline{X}} = \exp[-(E_X + E_{\overline{X}})] = \exp[-(E_\psi + E_{\overline{\psi}})] = f_\psi^{eq} f_{\overline{\psi}}^{eq}$ Now the Boltzmann equation becomes

$$\begin{aligned} \frac{dn}{dt} + 3Hn &= -\langle\sigma|v_{rel}|\rangle[n^2 - n_{eq}^2] \\ \frac{dY}{dx} &= -\frac{x\langle\sigma|v_{rel}|\rangle s}{H(m)}(Y^2 - Y_{eq}^2) \end{aligned} \tag{31}$$

Where $Y = n/s = \overline{n}/s$ is the actual number of ψ and $\overline{\psi}$ per unit volume, and $Y_{eq} = n_\psi^{eq}/s = n_{\overline{\psi}}^{eq}/s$. Now

$$\begin{aligned} \langle\sigma|v_{rel}|\rangle &= (n_{eq})^{-2} \int d\Pi_\psi d\Pi_{\overline{\psi}} d\Pi_X d\Pi_{\overline{X}} \times (2\pi)^4 \times \\ &= \delta^4(p_\psi + p_{\overline{\psi}} - p_X - p_{\overline{X}}) \times |M|^2 \exp -(E_\psi/T) \exp -(E_{\overline{\psi}}/T) \end{aligned} \tag{32}$$

is the thermal averaged cross section. For non-relativistic case, i.e., $x >> 3$

$$Y_{eq} = \frac{45}{(2\pi)^4}(\pi/8)^{1/2}\frac{g}{g_{*S}}x^{3/2}\exp -x\,. \tag{33}$$

For relativistic case, i.e., $x << 3$

$$Y_{eq} = 0.278\frac{g_{eff}}{g_s} \tag{34}$$

$g_{eff} = g$ (for bosons) and $g_{eff} = \frac{3g}{4}$ (for fermions)

5.2 Hot Relics

A species that decouples when it is relativistic is often called hot relics. Freeze out occurs when $x_f \leq 3$. From 34 we see $Y_{\rm eq}$ is constant in this case. And the asymptotic value $Y_{x\to\infty}$ can be taken as its equilibrium value at freeze out.

$$Y_\infty = Y_{eq}(x_f) = 0.278\frac{g_{eff}}{g_{*s}(x_f)} \tag{35}$$

Considering the expansion remains isentropic thereafter, the abundance of ψ's today is

$$n_\psi = s_0 Y_\infty = 825\frac{g_{eff}}{g_{*s}(x_f)}cm^{-3} \tag{36}$$

With present entropy density of a co moving volume $s_0 = 2970cm^{-3}$, the present relic mass density contributed by a hot relic of mass m is

$$\rho_\psi = s_0 Y_\infty m = 2.97 \times 10^3 Y_\infty (m/eV)\text{eVcm}^{-3} \tag{37}$$

In terms of relative density

$$\Omega_\psi h^2 = \frac{\rho_\psi}{\rho_{cri}} = 7.83 \times 10^{-2}(g_{eff}/g_{*x_f})(\text{m/eV}) \tag{38}$$

$$\rho_{cri} = 1.879h^2 \times 10^4\text{eVcm}^{-3} \tag{39}$$

where $h = 0.7$ denotes the present value of Hubble parameter.

5.3 Cold Relic

A species that decouples when it was non-relativistic is called cold relic, i.e., $x_f \geq 3$ and Y_{eq} is exponentially decreasing with x as seen from Eq. 33. Here we need to parameterize the temperature dependence of annihilation cross section. In general thermal averaged velocity times $\langle\sigma|v_{rel}|\rangle \propto T^n$ where $n = 0$ for s-wave annihilation and $n = 1$ for p-wave annihilation. Therefore, $\langle\sigma|v_{rel}|\rangle = \sigma_0(T/m)^n = \sigma_0 x^{-n}$. With this parameterization the Boltzmann equation for the abundance becomes

$$\frac{dY}{dx} = -\frac{\sigma_0 s}{H(m)} x^{-n+1}(Y^2 - Y_{eq}^2)\,. \tag{40}$$

Substituting $H(m)$ and s we get

$$\frac{dY}{dx} = 0.264(g_*/g_{*s})m_{PL}m\sigma_0 x^{-2}(Y^2 - Y_{eq}^2)\,. \tag{41}$$

Long after decoupling Y_{eq} tracks Y very slowly, i.e $Y >> Y_{eq}$. With this approximation we get

$$Y_\infty = \frac{3.79(n+1)x_f^{n+1}}{(g_{*s}/g_*^{1/2})m_{PL}m\sigma_0} \tag{42}$$

Number density and relative energy density are calculated as in the case of hot relic

$$n_\psi = s_0 Y_\infty \;=\; 1.13 \times 10^4 \frac{(n+1)x_f^{n+1}}{(g_{*s}/g_*^{1/2})m_{PL}m\sigma_0} \tag{43}$$

$$\Omega_\psi h^2 = 1.07 \times 10^9 \frac{(n+1)x_f^{n+1}}{(g_{*s}/g_*^{1/2}) m_{PL}\sigma_0}$$
$$= 1.07 \times 10^9 \frac{(n+1)x_f}{(g_{*s}/g_*^{1/2})\langle\sigma|v|_{rel}\rangle m_{PL}} \tag{44}$$

From the above equations we can conclude that smaller the cross section, larger the relic abundance. It only depends on the annihilation cross section at freeze out. Here we are mostly interested in calculating s-wave annihilation ($n = 0$) which is independent of temperature [4].

6 The Scalar Singlet Model

Here we extend the SM by including a singlet scalar S and show that it can be a good candidate of CDM. The Lagrangian density of the model is given by:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm DM}\,. \tag{45}$$

We previously have assumed DM to be stable and it does not decay. so there should not be any odd term in $\mathcal{L_{DM}}$ including S. One way to achieve this is to claim that Lagrangian respects a Z_2 symmetry under which $S \to -S$ while all other particle goes to themselves. Thus $\mathcal{L}_{\rm DM}$ is given by:

$$\mathcal{L}_{\rm DM} = \frac{1}{2}(\partial_\mu S^\dagger)(\partial_\mu S) - \lambda_{SH}(H^\dagger H)(S^\dagger S) - \frac{1}{2}m_0^2(S^\dagger S) - \lambda_S(S^\dagger S)^2 \tag{46}$$

where λ_{SH} is the Dark Matter Higgs coupling and λ_S is the self coupling co-efficient of dark matter. [5]

After spontaneous breaking of $SU(2)_L \times U(1)_Y$ we have

$$H = \begin{pmatrix} 0 \\ \frac{h+v}{\sqrt{2}} \end{pmatrix} \tag{47}$$

$$H^\dagger H = \frac{h^2}{2} + \frac{v^2}{2} + hv \tag{48}$$

where $v = 246$ GeV. Now the effective Lagrangian is given by:

$$\mathcal{L_{DM}} = \frac{1}{2}(\partial_\mu S^\dagger)(\partial_\mu S) - \lambda_{SH}(\frac{h^2}{2} + \frac{v^2}{2} + hv)(S^\dagger S) - \frac{1}{2}m_0^2(S^\dagger S) - \lambda_s(S^\dagger S)^2 \tag{49}$$

In the effective theory, the mass term of S is given by:

$$m_D^2 = m_0^2 + \lambda_{SH}v^2 \tag{50}$$

We assume that the DM mass between 1GeV < $\mathrm{m_D}$ < 1TeV. In the early Universe the DM remains in thermal equilibrium through its interaction with the SM particles. In particular, $SS \to H^\dagger H$, $SS \to f\bar{f}$, $SS \to W^+W^-$ and $SS \to ZZ$ are relevant. As the temperature falls below its mass scale, it decouples from the thermal bath. The thermal averaged annihilation cross-section of S to SM particles in the above mentioned channels are given by:

$$\langle\sigma|v_{rel}|\rangle_{ff} = \sum_{f=fermions} \frac{\lambda_{SH}^2 m_f^2}{4\pi} \times \frac{(1-(m_f/m_D)^2)^{3/2}}{[4m_D^2 - m_h^2]^2 + m_h^2\tau_h^2} \tag{51}$$

$$\langle\sigma|v_{rel}|\rangle_{W^\pm} = \frac{2\lambda_{SH}^2 m_W^4}{8\pi m_D^2} \times (1-(m_W/m_D)^2)^{1/2} \times \frac{[1+\frac{1}{2}(1-2m_D^2/m_W^2)^2]}{[4m_D^2 - m_h^2]^2 + m_h^2\tau_h^2} \tag{52}$$

$$\langle\sigma|v_{rel}|\rangle_{ZZ} = \frac{2\lambda_{SH}^2 m_Z^4}{16\pi m_D^2} \times (1-(m_Z/m_D)^2)^{1/2} \times \frac{[1+\frac{1}{2}(1-2m_D^2/m_Z^2)^2]}{[4m_D^2 - m_h^2]^2 + m_h^2\tau_h^2} \tag{53}$$

$$\langle\sigma|v_{rel}|\rangle_{hh} = \frac{2\lambda_{SH}^2}{128\pi m_D^2} \times (1-(m_h/m_D)^2)^{1/2} \times [1 + \frac{m_h^2}{4m_D^2 - m_h^2} + \frac{2\lambda_{SH} v^2}{m_h^2 - 2m_D^2}]^2 \tag{54}$$

where τ_h is the Higgs decay width which is given by

$$\tau_h = \sum_{f=fermions} \frac{m_f^2}{8\pi v^2} \frac{(m_h^2 - 4m_f^2)^{3/2}}{m_h^2} + \frac{\lambda_{SH}^2 v^2}{8\pi} \frac{(m_h^2 - 4m_D^2)^{3/2}}{m_h^2} \tag{55}$$

Here the decay mode $h \to SS$ is allowed only if $m_D < m_h/2$. The details of analytical calculation is given in Appendix- F [7].

6.1 Relic Abundance of Dark Matter

We can calculate the relic abundance of DM S from the Boltzmann equation 31 discussed in section -4. Using the annihilation cross-sections (51,52,53,54) we get the relic density of S to be

$$\Omega h^2 = 1.07 \times 10^9 \frac{x_f}{g_{*s}/g_*^{1/2} m_{PL} \langle\sigma|v|_{rel}\rangle_{\text{tot}}} . \tag{56}$$

As discussed in section 3.1, g_* and g_* are same in early universe having value $g_* = 106.75$. The value Planck Mass is $m_{\mathrm{PL}} = 1.22 \times 10^{19}$ GeV. We take a typical freeze-out epoch x_f=20 . As there are two free parameters- λ_{SH} and m_D at the right hand side of the above equation, one can plot λ_{SH} versus m_D which satisfies the observed constraint: $0.1088 < \Omega h^2 < 0.1158$ [6]. Let us first check how the total annihilation cross-section varies as m_D for a typical value of

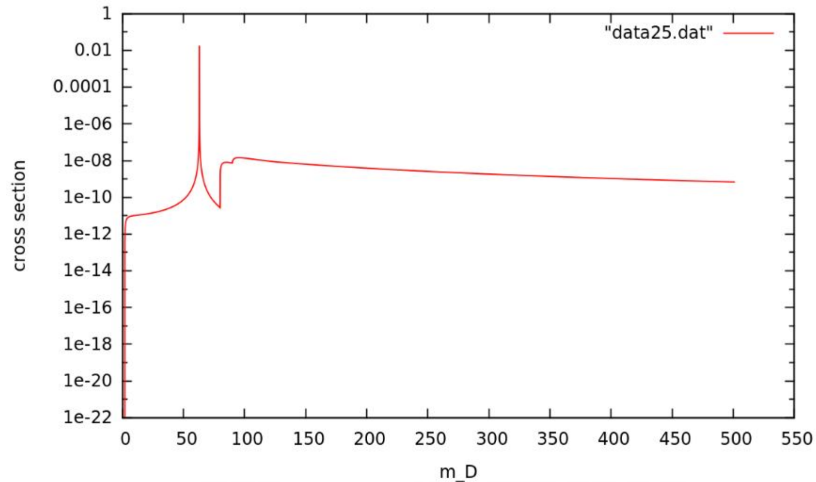

Figure 3: Thermal averaged total annihilation cross section $\langle\sigma|v|_{rel}\rangle$ vs mass of DM m_D(GeV) for $\lambda_{SH} = 0.1$.

λ_{SH}. From Fig-3, we see that there is a sudden rise in $\langle\sigma|v|_{rel}\rangle$ near $m_D \approx 63GeV = m_h/2$. This is the well known Bret-Wigner resonance arises due to the pole in the Higgs propagator appearing in the annihilation cross-section $SS \to ff$ (Eq. 51). As result the relic abundance decreases as $\Omega_{\rm DM}h^2 \propto 1/\langle\sigma|v|_{rel}\rangle$. However, this can be compensated by a small coupling λ_{SH} and can be seen from Fig-4. Outside the pole region the annihilation cross section decreases with increasing mass of DM as expected and shown in from Fig-3. Moreover, we also see from Fig-3 that small resonances at around $m_D = 80$ GeV and $m_D = 90$GeV. This is because of the pole of $SS \to ZZ$ annihilation at 80 GeV and $SS \to W^+W^-$ annihilation at 90 GeV. Thereafter $\langle\sigma|v|_{rel}\rangle$ starts decreasing with increase of m_D. Due to small resonances at $m_D = 80$ GeV and $m_D = 90$ GeV we expect the coupling to be small. Finally when the total cross-section decreases with increasing m_D, the coupling λ_{SH} start to increase slowly.

6.2 Discussion and Conclusion

With the discovery of the Higgs the SM of particle physics seems to be complete. However, it does not account the DM component of the Universe though we have compelling evidences against the existence of dark matter. Therefore, to include the DM component one has to go beyond the SM era. In this work after discussing the basics of early Universe thermodynamics we studied a simple extension of the SM to account DM abundance. We extended the SM by including a real singlet scalar and imposed a discrete Z_2 symmetry under which the DM

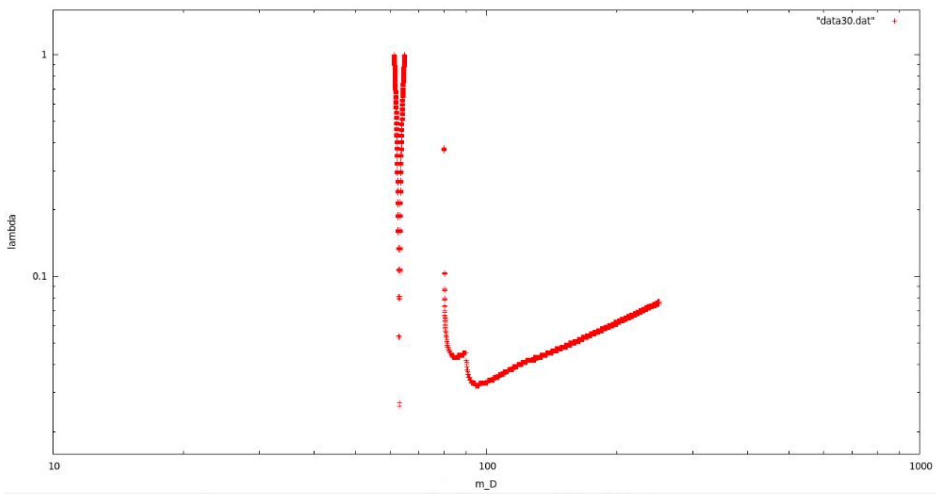

Figure 4: The contour in the plane of λ_{SH} versus m_D (GeV)for the observed relic abundance $0.1088 < \Omega h^2 < 0.1158$ [6].

is odd while all other particles are even. We showed the details of parameter space in which the real singlet can be a good candidate of dark matter. These parameter space can be verified from several direct detection experiments such Xenon-1T, Lux etc.

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1 Appendix

1.1 A

The non-zero components of Ricci tensor for R-W metric are

$$R_{00} = -3\frac{\ddot{a}}{a} R_{ij} = -[\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a} + 2\frac{k}{a^2}]g_{ij} \tag{1}$$

and Ricci scalar is given by,

$$\mathcal{R} \;=\; -6[\frac{\dot{\ddot{a}}}{a} + \frac{\dot{a}^2}{a} + \frac{k}{a^2}] \tag{2}$$

1.2 B

$$\begin{aligned} \rho &= \frac{g}{(2\pi)^3}\int_m^\infty d^3p f(E,t)E \\ \rho &= \frac{g}{2\pi^2}\int_m^\infty dE \frac{(E^2-m^2)^{1/2}E^2}{\exp[\frac{E-\mu}{T}]\pm 1} \end{aligned} \tag{3}$$

In the relativistic limit $T >> m$ and $T >> \mu$ the above equation becomes

$$\rho = \frac{g}{2\pi^2}\int_0^\infty dE \frac{E^3}{\exp E/T \pm 1} \tag{4}$$

using the general integration

$$\int_0^\infty dx \frac{x^{r-1}\exp(-ax)}{e^{-ax}-1} \;=\; \tau(r)\zeta(r)/a^r \tag{5}$$

$$\int_0^\infty dx \frac{x^{r-1}\exp -ax}{e^{-ax}+1} \;=\; \tau(r)(1-\frac{1}{2^{r-1}})\zeta(r)/a^r \tag{6}$$

$\zeta(r)$ is Reinmann-zeta function of order r and $\zeta(4) = \pi^4/90$ We get
$\rho = \frac{\pi^2}{30} g T^4$ (for bosons)
$\rho = \frac{7}{8}\frac{\pi^2}{30} g T^4$ (for fermions)
similarly

$$n = \;=\; \frac{g}{(2\pi)^3}\int d^3p f(E,t) \tag{7}$$

$$n = \frac{g}{2\pi^2}\int_m^\infty dE\frac{(E^3}{\exp[\frac{E-\mu}{T}]\pm 1} \tag{8}$$

using Eq.5 and 6 we get
$n = \zeta(3)\pi^2 gT^4$(for bosons)
$n = \frac{7}{8}(\zeta(3)\pi^2)gT^4$(for fermions)

$$p = \rho = \frac{g}{(2\pi)^3}\int_m^\infty d^3pf(E,t)\frac{|p|^2}{3E} \tag{9}$$

proceeding in the similar way,we get $p = \frac{\rho}{3}$ For non-relativistic case $m >> T$

$$n = \rho = \frac{g}{(2\pi)^3}\int_0^\infty d^3p\exp[\frac{E-\mu}{T}]\pm 1 \tag{10}$$

$E = (p^2+m^2)^{1/2} = m + p^2/2m + h.o$
since $m >> T$ $\exp\frac{E-\mu}{T}\pm 1 \approx \exp[\frac{E-\mu}{T}]$

$$\begin{aligned} n &= \frac{g}{2\pi^2}\exp{(m-\mu)/T}\int_0^\infty p^2\exp\frac{-p^2}{2mT} \\ &= g(\frac{mT}{2\pi})^2\exp[-(m-\mu)/T \end{aligned} \tag{11}$$

using the integral $\int_{-\infty}^{\infty} x^n\exp -x^2 dx = \Gamma(n+1)/2$

$$\begin{aligned} \rho &= \frac{g}{2\pi^2}\exp{(m-\mu)/T}\int_0^\infty p^2 E\exp\frac{-p^2}{2mT} \\ &= \frac{g}{2\pi^2}\exp{(m-\mu)/T}p^2(m+p^2/2m)\exp\frac{-p^2}{2mT}dp \end{aligned} \tag{12}$$

neglecting the $p^4/2m$ term,we get $\rho = nm$
proceeding in similar manner we get $p = nT$

1.3 C

$$TdS = d[(P+\rho)V] - VdP \tag{13}$$

also $TdS = \rho dV + Vd\rho + pdV$

$T(\frac{\partial S}{\partial V})_T = \rho + P$ [since ρ is not a function of V]

using Maxwell's relation$(\frac{\partial S}{\partial V})_T = (\frac{\partial P}{\partial T})_V$

$$T(\frac{\partial P}{\partial T})_V = (\frac{P+\rho}{T})dT \tag{14}$$

$$dP = \frac{(P+\rho)}{T} \tag{15}$$

Putting Eq. 15 into 13 we get

$$\begin{aligned} TdS &= d[(P+\rho)V] - V\frac{P+\rho}{T}dT \\ dS &= \frac{d(P+\rho)V}{T} - \frac{V}{T^2}(P+\rho)dT \\ &= d[\frac{(P+\rho)V}{T} + \text{constant}] \end{aligned} \tag{16}$$

Thus upto an additive constant entropy $S = V(P+\rho)/T$ Now,according to the first law of thermodynamics

$$\begin{gathered} dQ = 0 \\ d(\rho V) + PV = 0 \\ d[(P+\rho)V] = VdP \\ d[(P+\rho)V] = \tfrac{V}{T}(P+\rho)dT \\ d[\tfrac{(P+\rho)V}{T}] = \tfrac{V}{T^2}(P+\rho)dT \\ d[(P+\rho)V/T] = 0 \\ S = (P+\rho)V/T \end{gathered}$$

1.4 D

Boltzmann equation$\hat{L}[f] = C[f]$,where

$$\hat{L}f(E,t) = E\frac{\partial f}{\partial t} - \frac{\dot{a}}{a}|\vec{p}|^2\frac{\partial f}{\partial E} \tag{17}$$

using $|p| \approx E$

$$\frac{c[f]}{E} = \frac{\partial f}{\partial t} - \frac{\dot{a}}{a}E\frac{\partial f}{\partial E}$$

$$
\begin{aligned}
\frac{g}{(2\pi)^3}\int d^3p\frac{c[f]}{E} = \frac{g}{(2\pi)^3}\int d^3p[\frac{\partial f}{\partial t} - \frac{\dot{a}}{a}E\frac{\partial f}{\partial E}] \\
= \frac{g}{(2\pi)^3}[\int d^3p\frac{\partial f}{\partial t} - 4\pi\int dE\frac{\dot{a}}{a}E^3\frac{\partial f}{\partial E}]
\end{aligned} \quad (18)
$$

By integrating by parts the above equation becomes and using $n = \frac{g}{(2\pi)^3}\int d^3pf$

$$
\frac{g}{(2\pi)^3}\int d^3p\frac{c[f]}{E} = \frac{dn}{dt} - \frac{g}{(2\pi)^3}\times 4\pi[E^3f|_{boundary} - 3\int dEE^2f] \quad (19)
$$

Now, f goes zero at the boundary and we get

$$
\frac{g}{(2\pi)^3}\int d^3p\frac{c[f]}{E} = \frac{dn}{dt} + 3Hn \quad (20)
$$

1.5 E

Friedmann equation

$$
\begin{aligned}
H^2 = \frac{8\pi\rho G}{3} \\
H = \sqrt{\frac{8\pi G\rho}{3}}
\end{aligned} \quad (21)
$$

using $\rho = \frac{\pi^2 g_*^2 T^4}{30}$ and $m_{PL} = \sqrt{\hbar c/G}$ $H = 1.66\frac{T^2}{m_{PL}}g_*^{1/2}$ and for radiation dominated epoch $a \propto t^{1/2}$ $t = 0.301g_*^{-1/2}\frac{m_{PL}}{T^2}$

1.6 F

1.7 $SS \to f\overline{f}$ channel

amplitude

$$
iM = iv\frac{\lambda_{SH}}{(p+\acute{p})^2 - m_h^2 + im_h\tau_h}\overline{u}(\acute{k},\acute{s})(ig)v(k,s) \quad (22)
$$

$$
|M|^2 = \frac{v^2\lambda_{SH}^2 g^2}{(4m_D^2 - m_h^2)^2 + m_h^2\tau_h^2}\sum_{s,\acute{s}}\overline{u}(\acute{k},\acute{s})v(k,s)u(\acute{k},\acute{s})v(k,s) \quad (23)
$$

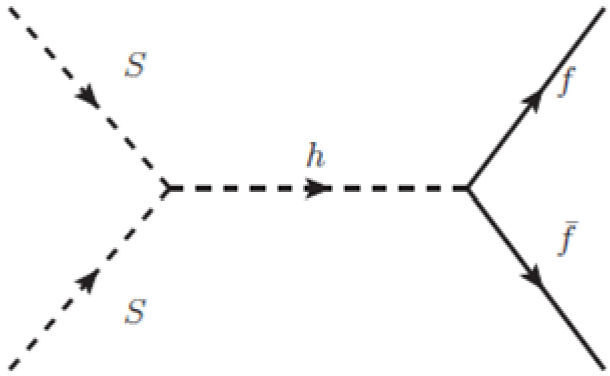

Figure 1: Feynman diagram for the process$SS \to f\overline{f}$ for S having momenta$p, \acute{p}$ and $f, \overline{f}$ having momenta $k, \acute{k}$

using completeness relations of Dirac equation

$$\sum_s u(p,s)\overline{u}(p,s) = \not{p} + m \tag{24}$$

$$\sum_s v(p,s)\overline{v}(p,s) = \not{p} - m \tag{25}$$

we get

$$|M|^2 = \frac{v^2\lambda_{SH}^2 g^2}{(4m_D^2 - m_h^2)^2 + m_h^2\tau_h^2} Tr[(\not{k} + m_f)(\not{k} - m_f)] \tag{26}$$

$$Tr[(\not{k} + m_f)(\not{k} - m_f)] = 4(\acute{k}.k - m_f^2) \tag{27}$$

$$\acute{k}.k = E^2 + |k|^2 = 2E_f^2 - m_f^2 = E_{cm}^2/2 - m_f^2 = 2m_D^2 - m_f^2 \tag{28}$$

Differential cross section

$$\frac{d\sigma}{d\Omega}|_{CM} = \frac{1}{E_{cm}^2|v_{rel}|} \times \frac{|k|}{16\pi^2 E_{cm}}|M|^2 \frac{v^2\lambda_{SH}^2 g^2}{16\pi^2 m_D^3|v_{rel}|} \tag{29}$$

$$= \times \frac{(m_D^2 - m_f^2)^{3/2}}{[(4m_D^2 - m_D^2)^2 + m_h^2\tau_h^2} \tag{30}$$

$$\sigma_{tot} = \int d\Omega \frac{d\sigma}{d\Omega} \tag{31}$$

From Yukawa coupling we know that $m_f = \frac{gv}{\sqrt{2}}$.Thus,summing over all fermion channels

$$\langle \sigma |v_{rel}| \rangle = \sum_{f=fermions} \frac{\lambda_{SH}^2 m_f^2}{4\pi} \times \frac{(1-(m_f/m_D)^2)^{3/2}}{[4m_D^2 - m_h^2]^2 + m_h^2 \tau_h^2} \tag{32}$$

1.8 $SS \to W^+W^-$channel

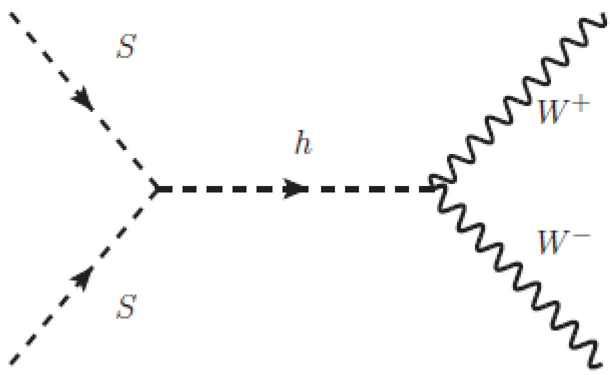

Figure 2: Feynman diagram for $SS \to W^+W^-$. SS having momentum $p, \acute{p}$ and W^+W^- having momentum $k, \acute{k}$

Here the coupling of Higgs with $W^{\pm}$ is given by SM as $g^2v/2$

$$\begin{aligned} iM &= iv\lambda_{SH} \frac{i}{4m_D^2 - m_h^2 + im_h\tau_h} \epsilon_\mu^*(\acute{k})(ig^2v/2)\epsilon_\mu(k) \\ |M|^2 &= \frac{\lambda_{SH}^2 g^4 v^4}{4(4m_D^2 - m_h^2)^2 + m_h^2\tau_h^2} \sum_{allpolarization} \epsilon_\mu^*(\acute{k})\epsilon_\mu(k)\epsilon_\nu^*(k)\epsilon_\mu(\acute{k}) \end{aligned} \tag{33}$$

Using the completeness relation

$$\sum_{allpolarization} \epsilon_\mu(p)\epsilon_\nu^*(p) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m_W^2} \tag{34}$$

And proceeding in the similar way we did in section 2.1

$$|M|^2 = \frac{v^2\lambda_{SH}^2 g^4 v^2}{4(4m_D^2 - m_h^2)^2 + m_h^2\tau_h^2}[2 + \frac{(2m_D^2 - m_W^2)^2}{m_W^4}] \tag{35}$$

In terms of mass of W bosons $m_w = gv/2$ and using equations 29,31 we finally get

$$\langle\sigma|v_{rel}|\rangle_{W^\pm} = \frac{2\lambda_{SH}^2 m_W^4}{8\pi m_D^2} \times (1-(m_Z/m_D)^2)^{1/2} \times \frac{[1+\frac{1}{2}(1-2m_D^2/m_W^2)^2]}{[4m_D^2-m_h^2]^2+m_h^2\tau_h^2}$$

1.9 $SS \rightarrow ZZ$channel

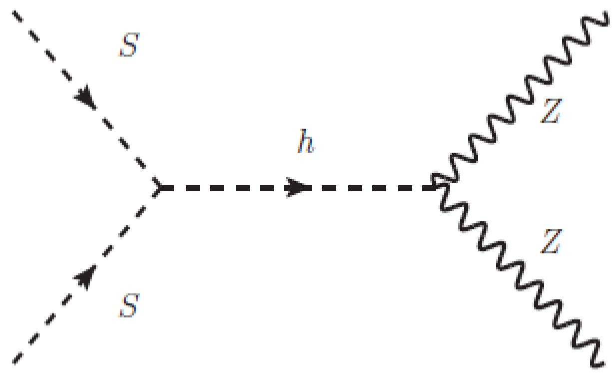

Figure 3: Feynman diagram for $SS \rightarrow ZZ$. SS having momentum $p, \acute{p}$ and ZZ having momentum $k, \acute{k}$

Here the calculation is as same as the previous came.Except here the coupling of Higgs with Zbosons is $g^2 v\cos^2\theta_w$, and $m_Z = \frac{gv\cos\theta_w}{2\sqrt{2}}$.

$$\langle\sigma|v_{rel}|\rangle_{ZZ} = \frac{2\lambda_{SH}^2 m_Z^4}{16\pi m_D^2} \times (1-(m_Z/m_D)^2)^{1/2} \times \frac{[1+\frac{1}{2}(1-2m_D^2/m_Z^2)^2]}{[4m_D^2-m_h^2]^2+m_h^2\tau_h^2} \tag{36}$$

1.10 $SS \rightarrow hh$ channel

Here three possible channels are there.

Here the self-coupling between Higgs is given by the co-efficient of h^3 term in Higgs potential $\mu^2 2(v+h)^2 + \frac{\lambda}{4}(v+h)^4$ which is in terms of Higgs mass $-\frac{m_h^2}{2v}$ The amplitude term

$$iM_1 = iv\lambda_{SH}(i)\frac{-im_h^2}{2v(4m_D^2-m_h^2)} \tag{37}$$

$iM_2 = i\lambda_{SH}v\frac{i}{[(p-k)^2-m_D^2}]^2$
$(p-k)^2 \approx m_h^2 - m_D^2$

$iM_3 = i\lambda_{SH}/2$ Now total amplitude for the process becomes

$$iM = i(M_1+M_2+M_3) \tag{38}$$

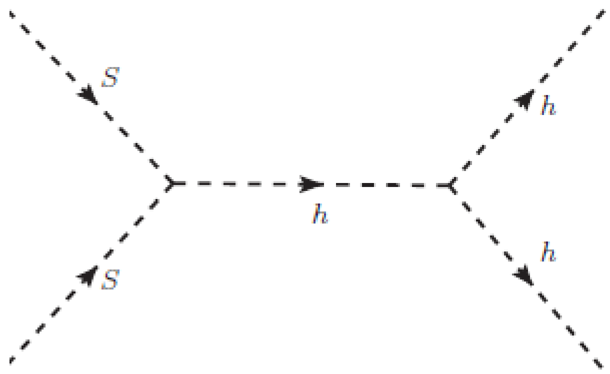

Figure 4: Feynman diagram for SS going to hh via higgs

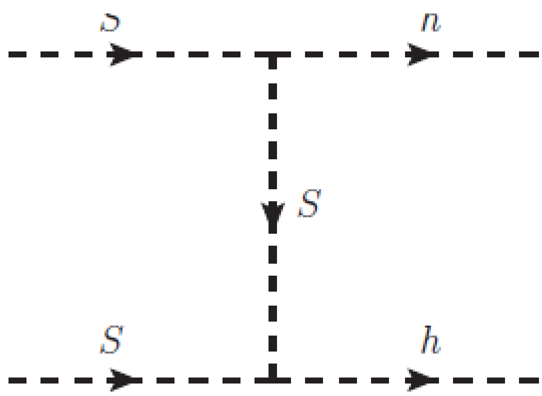

Figure 5: Feynman diagram for SS going to hh via S. SS having momentum $p, \acute{p}$ and h, h having momentum $k, \acute{k}$

proceeding in the similar way as in previous sections we get

$$\langle\sigma|v_{rel}|\rangle_{hh} = \frac{2\lambda_{SH}^2}{128\pi m_D^2} \times (1-(m_h/m_D)^2)^{1/2} \times [1+\frac{m_h^2}{4m_D^2-m_h^2}+\frac{2\lambda_{SH}v^2}{m_h^2-2m_D^2}]^2 \quad (39)$$

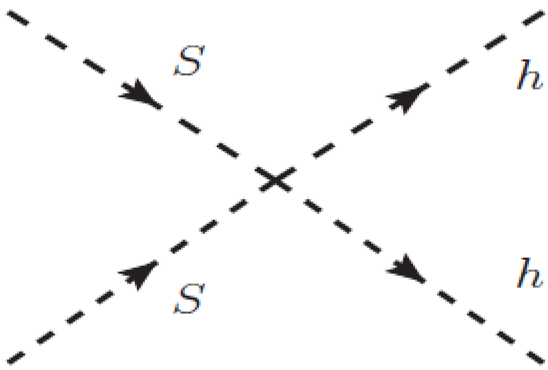

Figure 6: This diagram resembles to the four fermi interaction